RAN-2103000206020036

B. Sc. (Sem. - VI) Examination October - 2023

Mathematics - MTH-606

Number Theory - II

સૂચના : / Instructions

(૧) નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **B. Sc. (Sem. - VI)**

Name of the Subject :

☛ **Mathematics - MTH-606 Number Theory - II**

Subject Code No.: **2103000206020036**

Seat No.:

Student's Signature

(2) All questions are compulsory.
(3) Follow usual notations.
(4) Figures to the right indicate marks of the question.
(5) Total marks 50.

Q. 1 Answer any FIVE as directed. (10)

(1) Define *Pseudo prime*. Also give an illustration.

(2) If n and $n + 2$ are primes, then prove that $\sigma(n + 2) = \sigma(n) + 2$.

(3) Find the value of $\tau(360)$ and $\sigma(360)$.

(4) Obtain the highest power of 7, which divides 100!

(5) If p and $2p + 1$ are both odd primes, then prove that

$\phi(n + 2) = \phi(n) + 2$ for $n = 4p$.

(6) Find the value of $\sum_{k=1}^{25} \mu(k!)$.

(7) Prove that $\left[\frac{n}{2}\right] - \left[-\frac{n}{2}\right] = n$; for all $n \geq 0$.

(8) If $a \equiv b(mod\ n)$ and $c \equiv d(mod\ n)$, then prove that $a + c \equiv b + d(mod\ n)$.

Q. 2 Attempt any TWO. (10)

(1) Obtain three consecutive integers, the first is divisible by a fourth power, the second by a cube and third by square.

(2) Solve the Diophantine equation $12x + 25y = 331$ using congruencies.

(3) Find the integers having the remainders 2, 3, 4, 5, when divided by 3, 4, 5 and 6 respectively.

Q. 3 Attempt any TWO. (10)

(1) State and Prove : Fermat's theorem.

(2) If "p" is a prime, then prove that $a^p \equiv a \pmod p$ for any integer "a".

(3) If p is a prime then for any integer a, prove that

(i) $p | a^p + (p-1)!a$ and

(ii) $p | (p-1)!\, a^p + a$

Q. 4 Attempt any TWO. (10)

(1) If $n = p_1^{k1} \cdot p_2^{k2} \cdot ...p_r^{kr}$ is the prime factorization of $n > 1$, then prove that

(a) $\tau(n) = (k_1 + 1)(k_2 + 1) \ldots (k_r + 1)$ and

(b) $\sigma(n) = \left(\frac{p_1^{k_1+1} - 1}{p_1 - 1}\right) \left(\frac{p_2^{k_2+1} - 1}{p_1 - 1}\right) \ldots \left(\frac{p_r^{kr+1} - 1}{p_r - 1}\right)$

(2) Prove that the Möbius function μ. is multiplicative function.

(3) Find the highest power of 1000! terminates in 249 zeroes.

Q. 5 Attempt any TWO. (10)

(1) If p is a prime and $k \geq 1$, then prove that $\phi(p^k) = p^k - p^{k-1}$.

(2) For any odd integer a if 5|a, then prove that a and a^{4n+1} have the same last digit.

(3) For integers a, b, c; prove that gcd(a, bc) = 1 if and only if gcd(a, b) = 1 and gcd(a, c) = 1.
